Development of Bi-directional Two-Stage Transport Matrix Method for Strip Profile and Shape Calculation

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Crown, Shape, Rolling Mill, Mathematical Model, Transport Matrix, Strip Profile, Roll Shifting, Roll Bending, Roll Crossing

Abstract
A mathematical model of strip profile and shape using bi-directional two-stage transport matrix method is proposed in this article. The strip profile can be solved by applying an equivalent spring-beam-gap model. The distance and force transport matrices co-relate rolling parameters into a simple and small-rank linear system. This method provides such a very efficient and fast solution as to be applied for the on-line control.

Introduction
Crown and shape control is one of the most important factors on strip quality, material yield, and mill operation. For past decades, engineers have invented many crown/shape control devices - from simple roll bending to sophisticated combination of roll bending, roll shifting, roll profiling, and roll crossing with special roll contours. Various mill configurations – from 2-hi to multi-hi mills – are also developed to associate with selected crown control devices. In order to coordinate so many crown control devices, numerous optimal crown/shape control methods were researched to optimize mill performance. Among this series of developments, mathematical models play key roles to calculate the profiles in the roll/roll and roll/strip interfaces of the mill.

The elastic foundation method [1] was introduced by Stone (1965)

Shohet and Townsen (1968), influence coefficient method

Kizake: Point match method

Turley lump system

McDermott nonlinearity

Guo removal of nonlinearity by using spring-beam-gap system

the influence coefficient method [2] are pioneer works in late 60's. Later developments of other methods focus on improvement of the basic theory [3-6] and the computing efficiency [7-8].

Two-stage transport matrix method is one of the most significant improvements in the crown/shape computation field [9]. Although the wave-front type computing scheme provides fast convergence of the solutions for most mills with symmetric operation (such as roll bending and roll crossing), this method is not sufficient for a mill with roll shifting mechanism and non-symmetric roll profiling due to non-symmetrical mill geometry.

This article proposes a novel idea to allow two wave fronts propagate from both ends of the mill. The wave fronts will travel half way to meet together at the mill center. The solutions of mill parameters at both ends of the mill can be obtained after combining the boundary conditions due to mill operation. The interface contours and the strip profile can then be calculated using the inverse transport matrices. The bi-direction transport method of mill parameters can shorten the computing time significantly so as to apply the model on line.

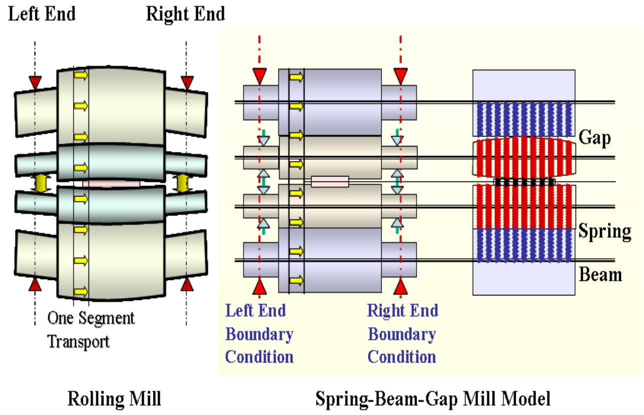

Figure 1: Spring-Beam-Gap Mill Model

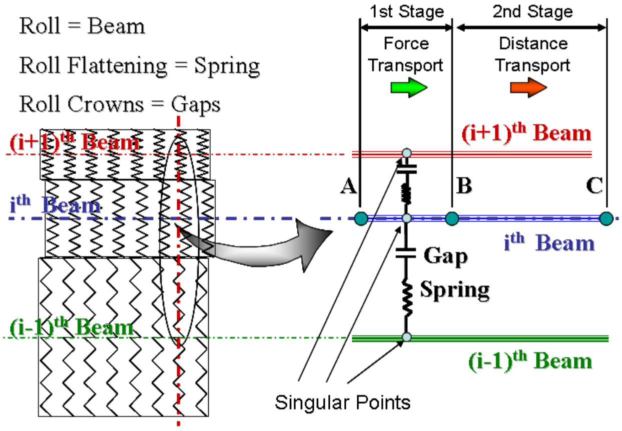

Figure 2: Two-Stage Transport Matrices

Two-Stage Transport Matrix Method

Two-stage transport matrix method was originated from the spring-beam-gap model. It was developed to solve the model with faster computing scheme. As shown in Figure 1, the roll is simulated by the beam element, the roll flattening is characterized by the spring element, and the roll/strip crowns are considered using gap elements. For each segment, the spring, beam, and gap elements of all rolls are placed in line. The physical variables – shear force V, bending moment M, slope θ, and displacement y – of two adjacent segment can be correlated (transported) by a distance transport matrix. There is a particular transport matrix for any two segments in the model. For any roll configuration, there is a corresponding spring-beam-gap model with various segments.

Figure 2 shows that the spring connection points of each beam (roll) are singular points. Direct transport over the singularity will cause serious errors. The two-stage method adds one force transport stage (from A to B) to cross over the singularity. There is no deflection difference between nodes A and B due to the infinitesimal distance. The adjacent beams (rolls) reacts each other in this stage to transfer the force and moment. The second-stage transport includes a deflection difference over a longer distance from B to C. This transport focuses only on the ith beam, having no effects on other adjacent beams.

Successive transport from the left side to the right side of the mill can form a global transport matrix for the variables of two sides. A low-rank linear system can be obtained by joining the global matrix with the boundary conditions that are determined by the control modes.

Distance Transport Matrix

Figure 3 shows a finite length beam element subjected to a concentration P and distributed load q. According to the basic bending fourth order differential equation and shear deflection equation, the distance transport matrix D_{RL} (from B to C as shown in Figure 2) can be written as:

(1) $$H_R = D_{RL} H_L$$

where

$$H_R = [1 \quad V_R \quad M_R \quad -\theta_R \quad -y_R]^T$$
$$H_L = [1 \quad V_L \quad M_L \quad -\theta_L \quad -y_L]^T$$

$$D_{RL} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ T_1 & 1 & 0 & 0 & 0 \\ T_2 & L & 1 & 0 & 0 \\ T_3 & \frac{L^2}{2EI} & \frac{L}{EI} & 1 & 0 \\ T_4 & \frac{L^3}{6EI} & \frac{L^2}{2EI} & L & 1 \end{bmatrix}$$

$$T_1 = -P - qL$$
$$T_2 = -PL_n - \tfrac{1}{2} qL^2$$
$$T_3 = -(3PL_n^2 + qL^3)/6EI + K_f (P + qL)/GA$$
$$T_4 = -(4PL_n^3 + qL^4)/24EI + K_f (PL_n + qL^2)/GA$$

The subscript R and L denote the right and left side. A is the cross section area of the beam element, E is Young's modulus, G is shear modulus, I is the second moment of inertia, and K_f is the shear deflection factor. There are no loading conditions between nodes B and C, only the distance transport matrix is considered for each beam.

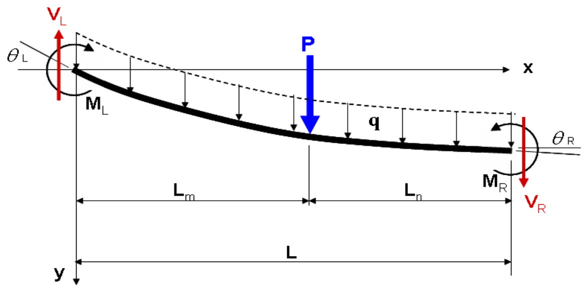

Figure 3: Formation of Distance Transport Matrix

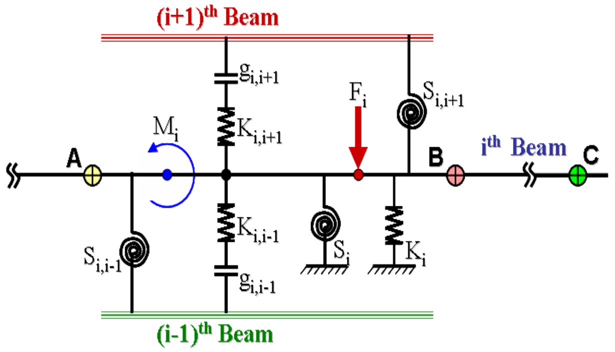

Figure 4 Formation of Force Transport Matrix

Force Transport Matrix

The force transport matrix (from A to B in Figure 2) can be derived from the chart as shown in Figure 4. There are many loading conditions in this region for three adjacent beams. The equilibrium equations for the (i-1)th, ith, and (i+1)th beams can be utilized to form the force transport matrix F_{ij} for the ith and jth beams.

For $|i\text{-}j|>1$, F_{ij} degenerates to a null matrix;

for $|i\text{-}j|=1$,

(2)
$$F_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ W_{ij} & 0 & 0 & 0 & k_{ij} \\ 0 & 0 & 0 & S_{ij} & 0 \\ -\beta_i W_{ij} & 0 & 0 & 0 & -\beta_i k_{ij} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and for $i=j$,

(3)
$$F_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ W_{ij} & 0 & 0 & 0 & -\sum k \\ -M_i & 0 & 0 & -\sum S & 0 \\ -\beta_i W_{ij} & 0 & 0 & 0 & -\beta_i \sum k \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$W_{ij} = \begin{cases} -k_{ij} g_{ij} + C_{ij} & for \quad j = i-1 \\ -k_i g_i + C_i - F_i & for \quad j = i \\ k_{ij} g_{ij} - C_{ij} & for \quad j = i+1 \end{cases}$$

$$\sum k = k_{i,i-1} + k_i + k_{i,i+1}$$

$$\sum S = S_{i,i-1} + S_i + S_{i,i+1}$$

C	Force intercept
F	Concentration force
g	Spring gap
k	Spring constant
M	External bending moment
S	Torsional spring constant
b	K/GA
i, j	between beams i and j
i	of beam i

Both individual distance and force transport matrices are *5 x 5* matrix, which can be assembled into a *5n x 5n* local matrix for each segment, where n is the total roll number. From sub-matrix viewpoint, the distance transport matrix is a diagonal matrix since there is no relationship between beams (rolls) during distance transport. The force transport matrix is a banded matrix with the bandwidth of 3 sub-matrices since the ith beam has the relations with the (i-1)th and (i+1)th beams only (see Eq. 3). For convenience, the symbols D_i and F_i are used hereafter to represent the distance and force transport matrices at the ith segment respectively.

The global transport matrix G can be obtained by multiplying all local distance and force transport matrices in sequence from left to right side of the mill:

(4)
$$\Phi_R = G_{RL} \Phi_L$$
where
$$\Phi_R = [H_{R1} \quad H_{R2} \quad .. \quad .. \quad .. \quad H_{Rn}]^T$$
$$\Phi_L = [H_{:L1} \quad H_{L2} \quad .. \quad .. \quad .. \quad H_{Ln}]^T$$

Equation (4) describes the relationship of physical variables between two ends of the mill. It is an under-determined linear system. Hence, it can be solved after associating with boundary conditions which depends on the mill control mode and the model type.

Boundary Conditions and Solutions

For a full model, the boundary conditions are located at both ends of the mill; for a quarter and half model, they are located in the left end and the center of the mill. The control mode – constant central line gage mode, constant total force mode, and constant roll gap mode – affects the formation of boundary conditions. At any cases, the boundary variables of V, M, q, y can be expressed by two vectors Φ_R and Φ_L.

After eliminating the dummy constant 1 in vector H (see Equation 1), the global transport matrix can be combined with the boundary conditions and the rigid body displacements of the rolls to form a 4(n+1) simultaneous linear equation set, which can be solved easily by Gaussian elimination method.

The transport matrix is derived from shear force, bending moment, slope, and displacement. There are large order differences between these four variables, for instance, the displacement has an order of O(-2) and the bending moment, O(7). Frequently, the original linear system leads to a large numerical error due to this large order difference. Hence, normalization of each variable becomes very critical of this method.

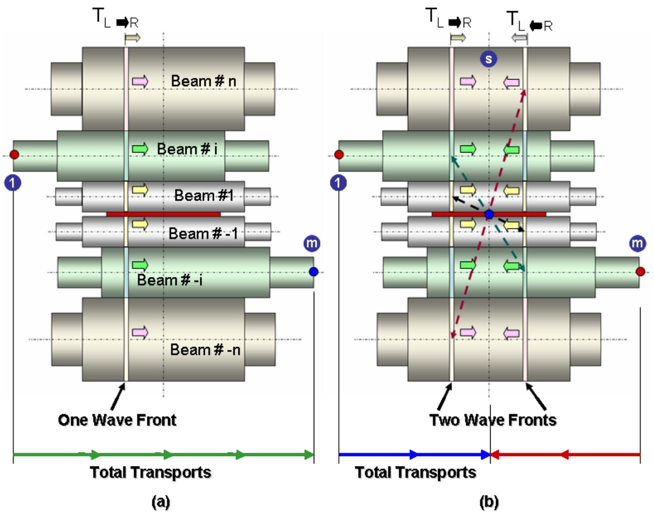

Figure 3: One- and Bi-directional Transports

Bi-Directional Transport Method

The one-directional transport method as described in the foregoing can save a great amount of computing time due to the small size of the matrix. For symmetric models, this method can provide a very fast solution. However, for asymmetric cases, such as roll shifting, this method needs to a long transport journey to reach the solution since it has only one wave front. Thanks to the asymmetric property, the local transport matrix of a particular segment at the left side should be very similar to its counterpart at the right side. For each wave propagation to the next segment, two local transport matrices can be generated and two wave fronts can be calculated simultaneously.

Figure 3 shows the basic transport concepts of two methods. Bi-directional method can save about 50% of computing time of generating and transporting matrices. Observing equation (4), the global transport matrix G_{RL} can be expressed by:

(5) $$G_{RL} = \prod_{i=1}^{i=m} D_i F_i = \left(\prod_{i=2}^{i=m} D_i F_i \right) D_1$$

Equation 5 shows that the global matrix is a product of all distance and force transport matrices from the 1st to the last segment (namely, the mth segment). Note that the first segment is always located at the roll neck area, there is no force transport matrix (F_1 is an identical matrix) due to no contact between any two roll necks.

The bi-directional method is applicable only if the roll stack can be mapped so that one and only one segment will contain the symmetric point (mostly the strip and mill center). This can be easily implemented by the designed program. Let the segment containing the asymmetric point (mostly the mill center) be the sth segment. Note that m must be an odd number and s=(m+1)/2. Equation (5) can be further rewritten in the following form:

(6) $$G_{RL} = \left(\prod_{i=2}^{i=m} D_i F_i \right) D_1 = D_m \left(\prod_{i=s}^{i=m-1} F_{i+1} D_i \right) F_s \left(\prod_{i=2}^{i=s-1} D_i F_i \right) D_1$$

Referring to Figure 3, the distance transport matrices D_m and D_1 are identical due to the same segment length and asymmetric property. Similarly, it is easy to derive that $D_j = D_{m+1-j} = D_{2s-j}$ with the very same reason. For the force transport, the corresponding matrices are similar, but not identical, namely, $F_j \approx \hat{F}_{2s+1-j}$. "Similar" means that the force transport matrix needs changing the orders of sub-matrices based on the roll positions. Consequently, Equation (6) can be further rewritten to show two wave fronts:

(7) $$\begin{aligned} G_{RL} &= D_1 \left(\prod_{i=s-1}^{i=2} \hat{F}_i D_i \right) F_s \left(\prod_{i=2}^{i=s-1} D_i F_i \right) D_1 \\ &= (D_1 \hat{F}_2 D_2 \cdots \hat{F}_{s-1} D_{s-1}) F_s (D_{s-1} F_{s-1} \cdots D_2 F_2 D_1) \\ &= T_{R>>L} F_s T_{L>>R} \end{aligned}$$

where $\hat{F}_i$ is the similar matrix to the matrix F_i. $T_{R>>L}$ is the transport matrix from the right to left side while $T_{L>>R}$, from left to right. Equation 7 shows that the local distance and force transport matrices can be easily obtained for both sides. Therefore, two wave fronts will move towards the center of the mill from both ends. Two wave fronts meet at the center of the mill and F_s plays the last step force transport to link two wave fronts.

Differences between Two Methods

As mentioned before, a quarter model considers top-left portion of the mill since the mill is fully symmetric on both x and y axis. The half model can be used for x- or y-symmetric mill in case that the mill has different top and bottom roll diameters (left-right symmetric) or wedge strip with mill tilting (top-bottom symmetric). The one-way transport method is suitable for both quarter and half models.

The roll shifting in an opposite direction will turn the mill into an asymmetric mill. Hence, the bi-directional transport method must be applied, particularly in the cases where the iteration is required. The iteration loop is necessary to check the roll opening. It is to make sure that all spring forces must be compressive forces, otherwise, no contact interface. Practically, as long as the roll crowns are not extremely large, the model can obtain the solution without any iteration.

However, the iteration loop is indispensable in case of strip shape calculation. It is because the strip shape affects the tension distribution which affects the rolling force that in turns affects the spring constant distribution of the strip. Therefore, although no re-mapping is required, the program still needs to iterate the same routine until the calculated strip shape is stabilized. The bi-directional method becomes much more important in this particular case. Yet, since there are no changes from the left edge of the model to the left edge of the strip, the transport matrix in this zone remains the same in all iterations. Equation (8) shows that the global transport matrix can be further split into two parts – the roll portion and the strip portion:

$$G_{RL} = T_{R>>L} F_s T_{L>>R} = (T_{R>>L})_{roll}\,(T_{R>>L})_{strip}\; F_s\,(T_{L>>R})_{strip}\,(T_{L>>R})_{roll} \tag{8}$$

$(T_{L>>R})_{roll}$ is the transport matrix from the left edge of the model to the left edge of the strip, which is the same for all iterations. And $(T_{L>>R})_{strip}$ is the transport matrix from the left edge of the strip to the center of the strip, which changes according to the strip shape distribution. Equation (8) provides an even faster computational routine for iterative procedures. However, if the strip shape is not symmetric to the strip center, the bi-directional method cannot be applied since the strip spring constant is no longer asymmetric due to various tension distribution. Hence, the full model with one-directional transport method should be adopted for non-symmetric strip shape cases.

In general, the cluster mill possesses a series of rolls with the intermediate roll shifting. For instance, the 20-hi cluster mill has 20 rolls with four shifting 1^{st} intermediate rolls. It takes much longer time to form the force and distance transport matrices due to its geometric complexity [10?]. The proposed bi-directional method should be the best choice to model the cluster mill.

Case Studies

Crown and shape control is one of the most important factors

Figure 6: Case 1: Roll Bending

Figure 7: Case 2: Roll Shifting

Figure 8: Case 3: Roll Bending + Roll Shifting

Discussion and Conclusion

1) Offline Application

2) Online Application

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